Can Everettian Interpretation Survive Continuous Spectrum?

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Abstract

This paper raises a simple continuous spectrum issue in many-worlds interpretation of quantum mechanics, or Everettian interpretation. I will assume that Everettian interpretation refers to many-worlds understanding based on quantum decoherence. The fact that some operators in quantum mechanics have continuous spectrum is used to propose a simple thought experiment based on probability theory. Then the paper concludes it is untenable to think of each possibility that wavefunction $\Psi\rangle$ gives probability as actual universe. While the argument that continuous spectrum leads to inconsistency in the cardinality of universes can be made, this paper proposes a different argument not relating to theoretical math that actually has practical problems.

1 Introduction

It is assumed that the Everettian interpretation of quantum mechanics refers to understanding based on quantum decoherence, first pursued by Everett (1957) [2] and later by the papers such as Zurek (1981) [4]. To say simply, in this interpretation, wavefunction $\Psi\rangle = \sum_k a_k\, k\rangle$ with $\sum_k |a_k|^2 = 1$ interacts with environment and reduced density matrix that recovers classical probability in terms of Born's rule is recovered when tracing over environment from original pure density matrix. By this, one can interpret each $k\rangle$ as being possible universe, with probability of being in the world k given by $|a_k|^2$, Born's rule. In this paper, a universe would refer to a single possible world allowed by the Everettian interpretation. Thus a universe does not refer to the set of worlds.

The interpretation has clear advantage of not having to think about how our world becomes chosen by probability mechanism, as one can simply say that wavefunction, or "decohered" wavefunction represents the state of entire universes.

But is this advantage really tenable? When spectra of operators are all countable, this seems to be reasonable, as countable additivity of probability theory is there - this will be discussed further in the later section. However, whenever we allow spectra of operators to be uncountable, thinking of $k\rangle$ as a different universe is highly problematic. Unless one assumes that there are some worlds privileged, the probability theory thought experiment demonstrates that the Everettian interpretation cannot match with what quantum mechanics predicts.

2 Continuous spectrum

It is known that integral must be used when trying to express Ψ in terms of position state vectors: $\Psi\rangle = \int_k a_k k\rangle dk$. The fact that any rigged Hilbert space that describes a quantum system is a subspace of some countable Hilbert space is therefore maintained, as one can always resort to a "good" basis. But when one wishes to think wavefunction in terms of universes realized, different possible positions should mean different universes - there is no intuitive reason why a good basis is "preferred basis" for describing different realities.
This immediately raises the question of cardinality, since we have $2^{\aleph_0}$ possible universes when described in possible positions, or $\aleph_0$ universes when described in terms of a good basis. In such a case, it is certainly possible to say that we instead have $2^{\aleph_0}$ universes, that share some "property" of the world. (If this is consistent understanding, then this proves that there needs to be at least $2^{\aleph_0}$ universes in the Everettian interpretation.) But as one will see, this understand has a fundamental problem.
Another possible solution would be to say that only countably many positions of all possible positions are realized as universes by picking the set of computable reals (for simplicity, I assume that position is one-dimensional). But the real line is selected arbitrarily, and function $x \to x + k$ with $x, k \in \mathbb{R}$ with k being some arbitrary constant should not affect qualitative results. It is certainly possible to adjust k so that previously a non-computable real x is a computable real $x + k$. The remaining possibility is to say that countably many universes are picked randomly - but then the Everettian interpretation no longer holds advantage over traditional interpretations of quantum mechanics.

3 Probability to a set of universes

There are also issues related to how one may assign probability to a set of universes. The question mainly is, since each branch/"universe" is equal, should not we assign equal probability to every outcome, barring the fact that uniform distribution over a real line does not exist, necessitating the need to use an improper prior, in case of continuous possible outcomes? One may easily go around this by allowing several universes to share same outcome, but the question of assigning probability, and in particular how Born's rule is realized, remains.
For a small subset of papers available, Deutsch (1999) [1] provides information-theoretical argument for assigning probability, Zurek (2005) [5] discusses envariance and derives Born's rule by quantum decoherence arguments. Sebens et

al. (2014) [3] discusses self-locating uncertainty and also derives Born's rule.

4 Does infinite cardinality matter in physics?

Discussions in previous sections mentioned uncountable and countable cardinality. But in physics, it may be the case that as long as cardinality is infinite, different cardinalities do not matter. Rigorously, one does say that some countable basis spans a quantum system, but sans rigor one may say that position state vectors do span the system, albeit with use of integral. This can be true for traditional interpretations such as Copenhagen - though what Copenhagen interpretation actually means has been different from a physicist to a different physicist. However, I will show in the next section that for the Everettian interpretation, uncountable cardinality matters.

5 Probability thought experiment

Assume that wavefunction Ψ for a particle satisfies $|\Psi(x,0)|^2 = 1$ for $0 < x < 1$ and $\Psi(x,0) = 0$ for rest. A wavefunction that satisfies above can be developed by Fourier analysis - I will drop the discussion here. I will ignore time, as this is not important to the analysis below.
Let X be random variable for one-dimensional position x of the particle. Let $y = f(x) = x^2$, with Y being random variable for position squared.
The probability density function (pdf) for X is:
$p(x) = 1$ with support $(0, 1)$.
$x = f^{-1}(y) = \sqrt{y}$, with $dx/dy = 1/(2\sqrt{y})$. The pdf for Y is:
$g(y) = 1/(2\sqrt{y})$.
Notice that $g(y) \neq p(x)$. Let us now take the following thought experiment:

- Following the Everettian interpreation, each possible outcome is now an individual world.
- But because of probability necessity, there will be other worlds that share the same outcome (thus avoiding criticism that every world is equally likely).
- Each world is associated with particle position x.
- Now x^2 is calculated inside the world.
- But subsets of universes were assigned probability in the Everettian interpretation. Since the universes that share some position x' must have equal chance of occurring compared to the universes sharing a different position x', it must be the case that just applying function f should not change this probability, because one is only dealing with/transforming the value! We only replaced associated x with associated x^2, and f is bijective.

> Thus Y must still have uniform distribution. We know that this cannot be consistent with quantum mechanics results.

(Observable for position can be referred as $\hat{X}$, with observable for position squared being $\hat{Y} = \hat{X}^2$, which follows matrix multiplication/exponentiation. $\hat{Y}$ and $\hat{X}$ obviously commute.)

It is then clear that the problem with the Everettian universe is the use of uncountably many "distinct" universes. Continuum of real numbers cannot be captured back into ordinals - here, universes.

5.1 Additional mathematical observations

To repeat what was said above, the Everettian interpretation requires assigning probability on universes, that can be mapped to probability derived by wave-functions. If universes are all equally likely, then this fact should remain so when labels change from x to x^2.
Everettians may, however, argue that "all universes being equally likely" is "obviously true," since each universe has zero probability, in case of continuous x. Thus, one must discuss probability of universes being in the interval (a, b) for label x or in the interval (a^2, b^2) for x^2, say hypothetical Everettians. I will argue that this observation misses the below mathematical point.
First of all, intuition that all universes being equally likely lead to uniform distribution (when location is localized to $(0, 1)$) is not really wrong. To see why Y does not exhibit uniform distribution and why the paradox results from the Everettian interpretation, the below formula is helpful:

$$y_2 = (x_2)^2 = (x + k)^2 = x^2 + 2kx + k^2 = y + 2kx + k^2$$

Assume that k is some real number. For any real $x_2 - x = k$, however small it is, one can find that $y_2 - y = 2kx + k^2$. $2kx$ factor is dependent on x, implying that x increases, $2kx$ factor increases. Thus, when universes that were distinguished by position x change label to x^2, they actually have some universes "missing" according to Y, and these missing universes affect probability proportional to x each y maps to.
Thus, the intuition that all universes equally likely should map to uniform distribution is a correct one - and that changing labels of universes by a bijective function should not affect probability of universes.

5.2 Density argument

Everettians may say that at position x, there are not really separate universes but a continuum of utilities given by density. Does this argument the Everettian interpretation?
Recall that all arguments explored in this paper so far against the Everettian universe do not rely on distinct universes existing at position x. Rather, these arguments required that different x should have some representative universe

that is assigned some probability. And in order for the Everettian interpretation to work, probability on the set of representative universes still must be considered, that is shown to reduce to probability deduced from Ψ. Thus density argument still fails.

5.3 Can infinitesimals then save the Everettian interpretation?

Recall $y_2 = (x_2)^2 = (x+k)^2 = x^2 + 2kx + k^2 = y + 2kx + k^2$. If k being real caused the problem, what if one allows infinitesimals? That is, assume that some universe at $x = x_1 + dx$ is not at $x = x_1$. Then what was explored in the subsection "Additional mathematical observations" does not seem to work, and the Everettian interpretation seems to be saved.
But by symmetry then, it is natural to allow infinitesimals for probability too. Why should positions only be allowed infinitesimals?
When allowing infinitesimals for probability, $P(A) = P(B)$ where A is one universe and B is some universe can be discussed in a clean way. This allows an easy way to prove that when universes are all equally likely, this should remain so regardless of any label.

6 Conclusion

The remaining concern is that for some pdf $p(x)$, countable points may differ from $p(x)$ without really affecting probability of events. This, however, does not apply for difference between the $p(x)$ and $g(y)$ discussed in this paper. Thus this concern can be ignored safely.
From the thought experiment, the reasonable conclusion I derive is that Everettian interpretation needs to develop probability of events (for example, probability of position of a particle being in $0.5 < x < 0.7$) independently of universes. That is, the Everettian universe must somehow dispose of the concept of distinct universes. But if this is required, then it must be asked whether there are really advantages in adopting the Everettian interpretation. The main advantage of the interpretation comes from its ability to treat possible outcomes as universes. This half-equivalence does naturally imply necessity to assign probability to the set of universes. Alternatives may be developed, and the author hopes to see the progress in light of this thought experiment.

References

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